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Intergrid Transfer Operators for Biharmonic Problems Using Nonconforming Plate Elements on Nonnested Meshes

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1. Introduction

The aim of this paper is to construct a preconditioner for biharmonic problems using nonconforming plate element on nonnested meshes by additive Schwarz methods. The success of the methods depends heavily on the existence of a uniformly,or nearly uniformly, bounded decomposition of a function space in which the problem is defined, and intergrid transfer operators with certain stable approximation properties play an important role in the decomposition [**1, 2, 4, 5, 7, 8, 3**]. For the case when coarse and fine spaces are all nonconforming, a natural intergrid operator seems to be one defined by taking averages of the nodal parameters. We define an intergrid transfer operator for nonconforming plate elements in this natural way, discuss its stable approximation properties, and obtain the stable factor $(H/h)^{3/2}$. It is also shown that the stable factor cannot be improved. However, to get an optimal preconditioner, we need in general the stability with a factor C independent of mesh parameters H and h. Therefore, it cannot be used for that purpose. To obtain an optimal preconditioner for biharmonic problems using nonconforming plate elements on nonnested meshes by additive Schwarz methods, we define an intergrid transfer operator, prove certain stable approximation properties, construct a uniformly bounded decomposition for the finite element space, and then get optimal convergence properties with a not necessarily shape regular subdomain partitioning. Here the fine mesh may not be quasi-uniform.

2. A sharp estimate

Let Ω be a bounded polygonal domain in R^2 with boundary $\partial\Omega$. We consider the following biharmonic Dirichlet problem:

$$\Delta^2 u = f \text{ in } \Omega, u = \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega. \tag{1}$$

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The variational form of the problem (1) is : to find $u \in H_0^2(\Omega)$ such that

$$a(u,v) = (f,v), \forall v \in H_0^2(\Omega), \tag{2}$$

where

$$a(u,v) = \int_\Omega \{\Delta u \Delta v + (1-\sigma)(2\partial_{12}u\partial_{12}v - \partial_{11}u\partial_{22}v - \partial_{22}u\partial_{11}v)\}dx,$$

$$(f,v) = \int_\Omega fvdx, \sigma \in (0, 0.5) \text{ is the Poisson ratio .}$$

The unique solvability of the problem (2) for $f \in L^2(\Omega)$ follows from the continuity and coerciveness of the bilinear form in $H_0^2(\Omega)$ and Lax-Milgram theorem.

Let J_h be a triangulation of Ω, and V_h a nonconforming plate element space. The corresponding finite element discrete equation for problem (2) is : to find $u_h \in V_h$ such that

$$a_h(u_h, v_h) = (f, v_h), \quad \forall v \in V_h, \tag{3}$$

where

$$a_h(u_h,v_h) = \sum_{\tau \in J_h} \int_\tau \{\Delta u_h \Delta v_h + (1-\sigma)(2\partial_{12}u_h\partial_{12}v_h - \partial_{11}u_h\partial_{22}v_h - \partial_{22}u_h\partial_{11}v_h\}dx.$$

To obtain an optimal preconditioner of problem(3) by additive Schwarz methods, the intergrid transfer operator with certain stable approximation properties plays an important role. For the case when coarse and fine spaces are all nonconforming, a natural intergrid transfer operator seems to be one defined by taking the average of the values of the nodal parameters. We define an intergrid transfer operator in this natural way. However, it can be shown that the intergrid transfer operator is not suitable for obtaining an optimal preconditioner.

We take Morley element as an example. In this section, let J_H and J_h be two quasi-uniform triangulations of Ω and V_H the associated Morley element spaces. We assume that J_h is a refinement of J_H. Note that $V_H \not\subset V_h$.

The intergrid transfer operator $I_H^h : V_H \to V_h$ is defined as follows.

For $v \in V_H, I_H^h v \in V_h$ is defined so that

a) *if p is a vertex of J_h which is also a vertex of J_H or in the interior of $\tau \in J_H$, $(I_H^h v)(p) = v(p)$; for other vertices p of J_h, v may have a jump at p and $I_H^h v$ takes the averages of v at p;*

b) *if m is a midpoint of an edge of J_h which is in the interior of $\tau \in J_H$, $\frac{\partial(I_H^h v)}{\partial n}(m) = \frac{\partial v}{\partial n}(m)$; for one of other edge midpoints m associated with J_h, $\frac{\partial v}{\partial n}$ may have several jumps and $\frac{\partial(I_H^h v)}{\partial n}(m)$ takes the arithmetic average value of $\frac{\partial v}{\partial n}$ at m.*

About the operator I_H^h we have the following sharp estimates.

THEOREM 1. *For $v \in V_H$, we have*

$$|I_H^h v - v|_{0,h,\Omega} \le C(H^3 h)^{1/2} |v|_{2,H,\Omega}, \tag{4}$$

$$|I_H^h v - v|_{1,h,\Omega} \le C\left(\frac{H^3}{h}\right)^{1/2} |v|_{2,H,\Omega}, \tag{5}$$

and

$$|I_H^h v|_{2,h,\Omega} \le C\left(\frac{H}{h}\right)^{3/2} |v|_{2,H,\Omega}. \tag{6}$$

Furthermore, the estimates (5) and (6) are sharp.

The proof of the theorem can be found in Shi and Xie [**6**]. For other nonconforming plate elements, we can get similar results.

To get an optimal preconditioner, we need in general the stability with a factor C independent of mesh parameters H and h. Therefore, it cannot be used for obtaining an optimal preconditioner.

3. An intergrid transfer operator for nonconforming plate element on nonnested meshes

We now define another intergrid transfer operator, discuss its stable approximation properties, and construct an optimal precondotioner for problem(2) using nonconforming plate elements on nonnested meshes.

3.1. Stable approximation properties. Let J_{H_c} be a quasi-uniform triangulation of Ω. J_{H_c} will be referred to as the coarse grid. Here H_c is the maximum diameter of this coarse triangulation. Let J_h be a triangulation of Ω that satisfies the minimal angle condition in this section. In general,J_h is not a subdivision of J_{H_c}. We assume that each fine triangle intersects with at most n_0 coarse triangles,$n_0 \leq C$,

$$(7) \qquad \begin{cases} h \leq CH_c, \text{ and} \\ |\tau| \leq C|k|, \text{ if } \bar{k} \cap \bar{\tau} \neq \phi, \tau \in J_h, k \in J_{H_c}, \end{cases}$$

where $|\cdot|$ means the area in R^2.

Let V_{H_c} be Morley element spaces associated with meshes H_c, nodal parameters of which vanish on $\partial\Omega$. Note that $V_{H_c} \not\subset V_h$. For the space V_{H_c},we take $W_{H_c} = AR^{H_c}(\Omega)$ to be its conforming relative, where W_{H_c} is the P_5 Argyris element space $\{w \in C^1(\bar{\Omega}) : w|_T \in P_5(T), \forall T \in J_{H_c}, w = \partial_n w = 0 \text{ on } \partial\Omega\}$. The conforming interpolation operator $E_{H_c} : V_{H_c} \to W_{H_c}$ is defined as follows(cf. Brenner [**1**]):

$$(8) \qquad \begin{cases} (E_{H_c}v)(p) = v(p); \\ (D^\alpha E_{H_c}v)(p) = \text{ average of } (D^\alpha v_i)(p), |\alpha| = 1; \\ D^\alpha E_{H_c}v(p) = 0, |\alpha| = 2; \\ \partial_n E_{H_c}v(m) = \partial_n v(m); \end{cases}$$

where p is vertex,m is midpoint of sides,$v_i = v|_{T_i}$ and T_i contains p as a vertex. $E_h : V_h \to W_h$ can be defined similarly. We have [**1**]

$$(9) \quad \|v - E_{H_c}v\|_{L^2(T)} + H_c|v - E_{H_c}v|_{1,T} + H_c^2|E_{H_c}v|_{2,T} \leq CH_c^2|v|_{2,T}, \forall v \in V_{H_c},$$

where $T \in J_{H_c}$, and

$$(10) \quad \|w - E_h w\|_{L^2(\tau)} + h_\tau|w - E_h w|_{1,\tau} + h_\tau^2|E_h w|_{2,\tau} \leq Ch_\tau^2|w|_{2,\tau}, \forall w \in V_h,$$

where $\tau \in J_h$, h_τ is the diameter of τ.

Define the nodal interpolation operator $\Pi_{H_c} : C_0^1(\Omega) \to V_{H_C}$ as follows:

$$\begin{cases} \Pi_{H_c}v(p) = v(p), \\ \partial_n \Pi_{H_c}v(m) = \partial_n v(m). \end{cases}$$

$\Pi_h : C_0^1(\Omega) \to V_h$ can be defined similarly. Then, it is easy to prove that

(11)

$$\|v - \Pi_{H_c}v\|_{L^2(T)} + H_c|v - \Pi_{H_c}v|_{1,T} + H_c^2|\Pi_{H_c}v|_{2,T} \leq CH_c^2|v|_{2,T}, \forall v \in H^2(T),$$

where $T \in J_{H_c}$.

The intergrid transfer operator $I_{H_c}^h : V_{H_c} \longrightarrow V_h$ is defined by $I_{H_c}^h = \Pi_h \cdot E_{H_c}$. For nested meshes, certain stable approximation properties of the intergrid transfer operator were discussed in Brenner [**1**]. However, for nonnested meshes, it can not be proved in the same way. We have the following theorem, which plays an important role in our analysis.

THEOREM 2. *There exists a constant $C > 0$,independent of h, H_c such that for $u \in V_{H_c}$,*

$$|I_{H_c}^h u|_{2,h,\Omega} \le C|u|_{2,H_c,\Omega}, \tag{12}$$

$$\|u - I_{H_c}^h u\|_{0,\Omega} + H_c|E_{H_c}u - I_{H_c}^h u|_{1,h,\Omega} \le CH_c^2|u|_{2,H_c,\Omega}, \tag{13}$$

where $|u|_{i,H_c,\Omega}^2 = \sum_{T\in J_{H_c}} |u|_{i,T}^2$.

PROOF. We first prove (12). Let $\bar{u} = E_{H_c}u$. The essential step is to establish the estimate

$$|I_{H_c}^h u|_{H^2(k)}^2 \le C \sum_{\bar{\tau}\cap\bar{k}\neq\phi,\tau\in J_{H_c}} |\bar{u}|_{2,\infty,\tau}^2 |k|, \forall u \in V_{H_c}, \text{ here } k \in J_h. \tag{14}$$

Let $\tau = \Delta p_1p_2p_3$, and m_1, m_2, m_3 be the midpoint of the edge $\overline{p_2p_3}$, $\overline{p_3p_1}$, and $\overline{p_1p_2}$ of τ, respectively. If k belongs completely to a single coarse element $\tau, \tau \in J_{H_c}$, then (14) is obviously true. We now prove (14) in the case that k does not belong completely to arbitrary coarse element $\tau, \tau \in J_{H_c}$. We know that

$$|I_{H_c}^h u|_{H^2(k)}^2 \le C\sum_{i=1}^{3}(\partial_n(\bar{u} - \bar{u}_I)(m_i))^2, \tag{15}$$

where $\bar{u}_I$ means the linear interpolation of $\bar{u}$. Let $\overline{p_2m_1}$ be the line segment connecting points p_2 and m_1. We assume that $\overline{p_2m_1}$ is cut into l pieces by the coarse triangles $\tau_1^{p_2m_1}, \cdots, \tau_l^{p_2m_1}$,and $u(\cdot)$ is a polynomial on each piece. By the assumption made at the beginning of this section, $l \le C$. Therefore, by using the triangle inequality, we have

$$|\partial_n(\bar{u} - \bar{u}_I)(m_1))|^2 \le 2|\partial_n(\bar{u} - \bar{u}_I)(p_2)|^2 + 2|\partial_n\bar{u}(p_2) - \partial_n\bar{u}(m_1)|^2 \equiv I + II. \tag{16}$$

Let $g = \bar{u}-\bar{u}_I$, then $g(p_1) = g(p_2) = g(p_3) = 0$, and there exists $\xi_1 \in \overline{p_1p_2}, \xi_2 \in \overline{p_2p_3}$ such that

$$\partial_{\overline{p_1p_2}}g(\xi_1) = 0, \partial_{\overline{p_2p_3}}g(\xi_2) = 0. \tag{17}$$

Hence using the triangle inequality and the mean value theorem, we obtain

$$|\partial_{\overline{p_1p_2}}g(p_2)|^2 = |\partial_{\overline{p_1p_2}}g(p_2) - \partial_{\overline{p_1p_2}}g(\xi_2)|^2 \le C \sum_{\bar{\tau}\cap\bar{k}\neq\phi,\tau\in J_{H_c}} |\bar{u}|_{2,\infty,\tau}^2|k|, \tag{18}$$

and

$$|\partial_{\overline{p_2p_3}}g(p_2)|^2 = |\partial_{\overline{p_2p_3}}g(p_2) - \partial_{\overline{p_2p_3}}g(\xi_1)|^2 \le C \sum_{\bar{\tau}\cap\bar{k}\neq\phi,\tau\in J_{H_c}} |u|_{2,\infty,\tau}^2|k|. \tag{19}$$

From (16), (18)-(19) we have

$$I \le C \sum_{\bar{\tau}\cap\bar{k}\neq\phi,\tau\in J_{H_c}} |\bar{u}|_{2,\infty,\tau}^2|k|. \tag{20}$$

Similarly,

(21)

$$II = 2|\partial_n \bar{u}(p_2) - \partial_n \bar{u}(m_1)| \leq C \sum_{m=1}^{l} |\bar{u}|^2_{2,\infty,\tau_m^{p_2 m_1}} |k|, \leq C \sum_{\bar{\tau}\cap\bar{k}\neq\phi,\tau\in J_{H_c}} |\bar{u}|^2_{2,\infty,\tau} |k|.$$

For m_2, m_3,we can get the estimates similar to (16), (20) and (21). Therefore, from (16),(20) and (21) and their similar estimates for m_2 and m_3, we have

$$\sum_{i=1}^{3} |\partial_n(\bar{u} - \bar{u}_I)(m_i)|^2 \leq C \sum_{\bar{\tau}\cap\bar{k}\neq\phi,\tau\in J_{H_c}} |\bar{u}|^2_{2,\infty,\tau} |k|. \tag{22}$$

(14) follows from (15) and (22).

For $\tau \in J_{H_c}$,we denote by $\tau_j, j = 1, \cdots, l_1$,all the coarse triangles which share at least one of the fine triangles that intersects with τ(i.e.,this fine triangle intersects with both τ and τ_j). $l_1 \leq C$.

For all $k_i \in J_h, i = 1, \cdots, m'$,whose intersection with τ is nonempty, we obtain from (14) that

$$|I^h_{H_c} u|^2_{H^2(k_i)} \leq C \sum_{j=1}^{l_1} |\bar{u}|^2_{2,\infty,\tau_j} |k_i|, \forall u \in V_{H_c}. \tag{23}$$

By summing (23) over all $k_i, i = 1, \cdots, m'$, and from an elementwise inverse estimate we have

$$\begin{aligned} \sum_{k\cap\tau\neq\phi} |I^h_{H_c} u|^2_{H^2(k)} &\leq C \sum_{i=1}^{m'} \sum_{j=1}^{l_1} |\bar{u}|^2_{2,\infty,\tau_j} |k_i| \leq C \sum_{j=1}^{l_1} |u|^2_{2,\infty,\tau_j} \sum_{i=1}^{m'} |k_i| \\ &\leq C \sum_{j=1}^{l_1} |\bar{u}|^2_{2,\infty,\tau_j} |\tau| \leq C \sum_{j=1}^{l_1} |\bar{u}|^2_{2,,\tau_j}. \end{aligned} \tag{24}$$

Here we used the fact that,for each τ,the sum of the areas of the fine triangle that intersects with τ is less than $C|\tau|$ because of the assumption $h \leq CH_c$.

By summing (24) over all τ in J_{H_c} and noting that the number of repetitions,for each τ,in the summation is finite,we have

$$|I^h_{H_c} u|^2_{2,h,\Omega} \leq \sum_{\tau\in J_{H_c}} \sum_{\bar{k}\cap\bar{\tau}\neq\phi} |I^h_{H_c} u|^2_{H^2(k)} \leq C|\bar{u}|^2_{2,\Omega}. \tag{25}$$

(12) follows from (25) and (9).

We now turn to the proof of (13). Let $k \in J_h$ be a fine triangle and p one of its nodes,which implies that $w(p) = 0, \partial_n w(m) = 0$. Here $w = u - I^h_{H_c} u$. We consider the integral

$$\begin{aligned} \|w\|^2_{L^2(k)} &= \int_k w^2(x)dx \leq 2 \int_k (w(p) - w(x) - Dw(p).(x-p))^2 dx \\ &+ 2 \int_k |Dw(p).(x-p)|^2 dx \equiv I_1 + I_2. \end{aligned} \tag{26}$$

Let $\overline{xp}$ be the line segment connecting points x and p. We assume $\overline{xp}$ is cut into l_2 pieces by coarse triangles $\tau^k_1, \cdots, \tau^k_{l_2}$. Using the triangle inequality and the mean value theorem,we have

$$I_1 \le 2\int_k (w(p)-w(x)-Dw(p).(p-x))^2dx \le 2l_2\sum_{m=1}^{l_2}|w|^2_{2,\infty,\tau_m^k\cap k}h_k^4|k|, \tag{27}$$

where h_k is the diameter of element k. For the three edge midpoints $m_i(i=1,2,3)$ of k, and by using $\partial_n w(m_i)=0$ and the arguments similar to (17)-(19) we

$$I_2 \le 2l_2\sum_{m=1}^{l_2}|w|^2_{2,\infty,\tau_m^k\cap k}h_k^4|k|. \tag{28}$$

It follows from (26)-(28) and an elementwise inverse estimate that

$$\begin{aligned}\|w\|^2_{L^2(k)} &\le C\sum_{\bar\tau\cap\bar k\ne\phi,\tau\in V_{H_c}}|w|^2_{2,\infty,\tau\cap k}h_k^4|k|\\ &\le C\sum_{\bar\tau\cap\bar k\ne\phi,\tau\in V_{H_c}}(|\bar u|^2_{2,\infty,\tau}+|I^h_{H_c}u|^2_{2,\infty,k})h_k^4|k|\\ &\le C\sum_{\bar\tau\cap\bar k\ne\phi,\tau\in V_{H_c}}|\bar u|^2_{2,\infty,\tau}h_k^4|k|+Cn_0h^4|\bar u|^2_{2,k}.\end{aligned} \tag{29}$$

For $\tau\in J_{H_c}$,from (29) and by the same notation as (23) we have

$$\|w\|^2_{L^2(k_i)} \le C\sum_{j=1}^{l_1}|\bar u|^2_{2,\infty,\tau_j}h^4|k_i|+C|\bar u|^2_{2,k_i}h^4. \tag{30}$$

By summing (30) over all $k_i, i=1,\cdots,m$,and using the argument similar to (24) and the fact that $\sum_{k\cap\tau\ne\phi}|k|\le C|\tau|$ we have

(31)

$$\begin{aligned}&\sum_{\bar k\cap\bar\tau\ne\phi,k\in J_h}\|w\|^2_{L^2(k)}=\sum_{i=1}^{m'}\|w\|^2_{L^2(k_i)}\le C\sum_{i=1}^{m'}\sum_{j=1}^{l_1}|\bar u|^2_{2,\infty,\tau_j}h^4|k_i|+C\sum_{i=1}^{m'}|\bar u|^2_{2,k_i}h^4\\ &\le Ch^4\sum_{j=1}^{l_1}|u|^2_{2,\infty,\tau_j}\sum_{i=1}^{m'}|k_i|+Ch^4\sum_{j=1}^{l_1}|\bar u|^2_{2,\tau_j}\le Ch^4\sum_{j=1}^{l_1}|\bar u|^2_{2,\tau_j}.\end{aligned}$$

By summing (31) over τ in J_{H_c} and noting that the number of repetitions,for each τ,in the summation is finite,we obtain

$$\|w\|^2_{L^2(\Omega)}\le Ch^4|\bar u|^2_{H^2(\Omega)}\le Ch^4|u|^2_{H^2(\Omega)}, \tag{32}$$

and by the similar argument we can get

$$|w|^2_{1,h,\Omega}\le|E_{H_c}u-I^h_{H_c}u|^2_{1,h,\Omega}\le CH_c^2|E_{H_c}u|^2_{2,\Omega},\le CH_c^2|u|^2_{2,\Omega}. \tag{33}$$

Combining (32) and (33) completes the proof of the lemma. □

We now partition Ω into nonoverlapping subdomains $\{\Omega_i\}$,such that no $\partial\Omega_i$ cuts through any elements $\tau,\tau\in J_h$, and $\bar\Omega=\cup_{i=1}^N\bar\Omega_i$. Note that we do not assume that $\{\Omega_i\}$ forms a regular finite element subdivision of Ω,nor that the diameters of Ω_i are of the same order. To obtain an overlapping decomposition of Ω, we extend each Ω_i to a larger subdomain $\Omega_i'\supset\Omega_i$,which is also assumed not to cut any fine mesh triangles, such that $\mathrm{dist}(\partial\Omega_i'\cap\Omega,\partial\Omega_i\cap\Omega)\ge C\delta,\forall i$,for a constant $C>0$. Here $\delta>0$ will be referred to as the overlapping size. We assume that there exists an integer N_c independent of the mesh parameters h, H_c and δ such that any point in Ω can belong to at most N_c subdomains Ω_i'. For each $\Omega_i', i = 1, \cdots, N$,we define a finite element space

$$V_i = \{v \in V_h;\ \text{nodal parameters } = 0 \text{ at } \partial\Omega_i' \text{ and outside } \Omega_i'\}.$$

On the basis of Theorem 2 and (9)-(11), we can prove the next theorem, which shows that the decomposition $V_h = V_0 + V_1 + \cdots + V_N$ exists and is uniformly bounded when $\delta = O(H_c)$.

THEOREM 3. *For any $v \in V_h$,there exist $v_0 \in V_{H_c}, v_i \in V_i, i = 1, \cdots, N$, such that*

$$v = I_{H_c}^h v_0 + v_1 + \cdots + v_N,$$

and in addition,there exists a constant $C_0 > 0$,independent of the mesh parameters h, H_c and δ such that

$$a_{H_c}(v_0, v_0) + \sum_{i=1}^{N} a_h(v_i, v_i) \le C_0 N_c \left(1 + \frac{H_c^2}{\delta^2} + \frac{H_c^4}{\delta^4}\right) a_h(v, v), \forall v \in V_h.$$

The proof can be found in Xie [**7**].

3.2. An Additive Schwarz Method. Define $A_h : V_h \to V_h, A_i : V_i \to V_i (1 \le i \le N)$,and $A_{H_c} : V_{H_c} \to V_{H_c}$ by

$$\begin{aligned} (A_h v, w) &= a_h(v, w), \forall v, w \in V_h, \\ (A_i v, w) &= a_h(v, w), \forall v, w \in V_i, \\ (A_{H_c} v, w) &= a_{H_c}(v, w), \forall v, w \in V_{H_c}, \end{aligned}$$

respectively. The operator $Q_i : V_h \to V_i, 1 \le i \le N$, is defined by

$$(Q_i v, w) = (v, w), \forall v \in V_h, w \in V_i.$$

The operator $P_i : V_h \to V_i, 1 \le i \le N$,is defined by

$$a_h(P_i v, w) = a_h(v, w), \forall v \in V_h, w \in V_i.$$

The operators $I_h^{H_c}, P_h^{H_c} : V_h \to V_{H_c}$ are defined by

$$(I_{H_c}^h v, w) = (v, I_h^{H_c} w), \forall v \in V_{H_c}, w \in V_h,$$

and

$$a(I_{H_c}^h v, w) = a_h(v, P_h^{H_c} w), \forall v \in V_{H_c}, w \in V_h,$$

respectively.

The two level additive Schwarz preconditioner $B : V_h \to V_h$ is defined by

$$B := I_{H_c}^h A_{H_c}^{-1} I_h^{H_c} + \sum_{i=1}^{N} A_i^{-1} Q_i.$$

It can be easily seen that the operator $P = BA_h = I_{H_c}^h P_h^{H_c} + \sum_{i=1}^{N} P_i$ is symmetric positive-definite with respect to $a_h(\cdot, \cdot)$.

On the basis of Theorem 2 and Theorem 3, we obtain the following theorem which shows that P is uniformly bounded from both above and bellow when $\delta = O(H_c)$.

THEOREM 4. *The following estimate holds:*

$$\lambda_1 a_h(u,u) \le a_h(Pu,u) \le \lambda_2 a_h(u,u), \forall u \in V_h,$$

where

$$\lambda_2/\lambda_1 \le CN_c\left(1 + \frac{H_c^2}{\delta^2} + \frac{H_c^4}{\delta^4}\right),$$

which is independent of the diameter of subdomains. This allows us to use subdomains of arbitrary shape.

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