

## PREFACE

This volume captures the main part of the proceedings of the Seventh International Conference on Domain Decomposition Methods, which was hosted by The Pennsylvania State University, October 27–30, 1993. Over one hundred and fifty mathematicians, engineers, physical scientists, and computer scientists came to this nearly annual gathering – nearly half of them for the first time. Those attending from outside the United States accounted for about one-third of the registrants and came from 18 countries.

Since parallel sessions were employed at the conference in order to accommodate as many presenters as possible, attendees and non-attendees alike may turn to this volume for the latest developments. Most of the authors are to be commended for their efforts to balance the conflicting demands of writing for a diverse audience and staying within limits of twelve pages for invited lecturers and six pages otherwise. Enforcing page quotas was essential in accommodating the largest title count in the seven-volume history of the conference — selected from an even larger number of submissions.

After seven meetings, it would be natural to expect that domain decomposition methods would have moved into the mainstream, and therefore ceased to justify the special focus of their own conference. The interest of authors from many fields in entrusting final versions of their latest work to these proceedings supports this premise — while at the same time contradicting its conclusion! “Divide and conquer” domain decomposition may be the most basic of algorithmic paradigms, but theoreticians and practitioners alike are still seeking — and finding — incrementally more effective forms, and enjoy an interdisciplinary forum for them.

We comment briefly on the the term “domain decomposition” that has for nearly a decade been associated with this meeting and its proceedings. In the past few years, “domain decomposition” has become a synonym for “data parallelism” in the parlance of computer science, where it stands in contrast to “task parallelism.” In a generic sense, any algorithm that achieves concurrency by applying *all* of the operations independently to *some* of the data, as opposed to *some* of the operations independently to *all* of the data, may properly be called “domain decomposition,” but casting the net this broadly almost ceases to be useful. The PDE-motivated subject matter of this meeting has traditionally revolved around two foci within this very broadly defined class of algorithms: iterative subspace correction methods, and block elimination methods. In the former, which we may for convenience call “Schwarz methods” (though Schwarz’s recorded perspective was narrower), a domain solver is used as a subdomain solver inside an iteration over subdomains. In the latter, which have been classified “iterative substructuring,” and which we may in the same spirit call “Schur methods,” an operator equation for a lower-dimensional interface between subdomains is derived. Schwarz and Schur methods

may be unified in certain cases by regarding the Schwarz iteration as a map between iterates restricted to the interfaces. Both Schwarz and Schur approaches are now customarily used inside of a Krylov iteration (such as conjugate gradients), leading to what may be called “Krylov-Schwarz” and “Krylov-Schur” methods. These are terms that computer scientists are less likely to borrow.

The notion of a geometrical domain decomposition leads immediately to the notion of a function space decomposition, in which the subspaces are associated with subregions of support, and, in turn, to an operator decomposition, in which the relevant operators are restrictions of the original operator to the subspaces. This association between geometry and subspaces and operators allows many domain decomposition methods to be analyzed as iterative subspace correction methods, along with their relatives from classical iterative methods and multilevel methods, in which the decomposition of the corresponding function spaces is motivated by factors other than geometry.

Organizing the contents of an interdisciplinary proceedings is an interesting job, and our decisions will inevitably surprise a few authors, though we hope without causing offense. It is increasingly artificial to assign papers to one of the four categories of theoretical foundations, algorithmic development, parallel implementation, and applications, that are traditional for this proceedings series. Readers are encouraged not to take the primary divisions very seriously, but to trace all the connections.

Browsers turning to a preface expect a few words of context, particularly about what’s new. The volume-wide subject classifications, viz.,

- 65N55:** Multigrid methods; domain decomposition for BVPs
- 65N30:** Finite elements, Rayleigh-Ritz and Galerkin methods, finite methods
- 65F10:** Iterative methods for linear systems
- 65Y05:** Parallel computation
- 65M55:** Multigrid methods; domain decomposition for IVPs
- 65N35:** Spectral, collocation and related methods

give only a majority-weighted impression of the contents. Specific noteworthy trends in the Seventh International Conference on Domain Decomposition Methods are highlighted below.

- *elliptic PDE problems and progress in dealing with so-called “bad parameters”:*

For smooth problems, algorithms guaranteeing convergence rates that are asymptotically only weakly dependent of the size of the subdomains ( $H$ ) into which a domain is cut and the finest resolved length scale ( $h$ ) have been known for nearly a decade. The Poisson problem has inspired further theoretical work, primarily in establishing links to multilevel theory [Bank–Xu, Bornemann, Griebel]. The framework of “stable splittings” for iterative subspace correction methods is described in [Oswald], and permits discretization error estimates to be obtained as a by-product of algebraic convergence monitoring [Ruede]. The same benefit obtains from the cascade principle for the solution of general elliptic BVPs [Deuffhard]. The dependence of convergence rate on physical parameters such as jumps in the coefficients [Bakhvalov–Knyazev, Dryja, Le Tallec–Mandel–Vidrascu, Nepomnyaschikh] and irregular domain geometry not resolvable by a coarse grid [Kornhuber–Yserentant] have also come under study herein. Obstacle problems have been extended to

include first-order terms [Kuznetsov–Neittaanmaki–Tarvainen].

- *PDE developments outside of the elliptically-dominated framework:*

The problem of large first-order terms, which manifests itself in both discretization and solution phases of a PDE, has traditionally been handled by some form of elliptic domination. This is achieved by considering  $h$  sufficiently small, so that second-order derivative contributions to the stiffness matrix dominate first-order, or by artificially diffusive discretizations of the first-order terms in the operator to be inverted. (In this case, more accurate discretizations are typically used in the computation of the true residual). Recently, investigators have been working in from the other end of the Reynolds number spectrum, starting from methods that become all the more accurate as the elliptic terms become vanishingly small [Katzner, Layton–Maubach–Rabier]. (The approach in [Katzner] is related to frequency decomposition methods.) Preconditioners so derived may be much more efficient to apply than preconditioners coming from all of the terms of the discretization [Ashby–Kelley–Saylor–Scroggs]. Generalized Schwarz splittings, involving mixed (Robin-type) or tangential boundary conditions on artificial boundaries, with parameters dependent upon the advection seem promising [Tan–Borsboom], especially in the context of inexact subdomain solves, where parameters can be found that mitigate the loss of coupling in an overlapped block ILU technique [de Sturler]. “Outflow” boundary conditions are preferred in [Nataf–Rogier]. Meanwhile, the Schwarz alternating method has been generalized in another way, namely, overdetermined matching conditions within a layer of finite thickness [Sun–Tang] instead of well determined conditions along an edge. In addition to dealing with operator nonsymmetry and boundary conditions, research has continued into operator-splitting approaches for advection-diffusion. Such semi-implicit, semi-Lagrangian methods are able to exploit the method of characteristics for the pure advection part and the symmetric elliptic theory for the pure diffusion part [Chefter–Chu–Keyes, Wang–Dahle–Ewing–Lin–Vag]. Nonlinear problems, whose theory greatly lags practice, have come in for more theoretical attention during the past year [Cai–Dryja, Dawson–Wheeler, Tai]. (The ability to treat the full nonlinearity on the coarse grid only is studied in [Dawson–Wheeler].) Wave-Helmholtz problems are also considered [Kim].

- *non-traditional discretizations:*

Earlier volumes of this proceedings series were devoted almost entirely to low order discretizations based on conforming finite elements, finite differences, or finite volumes. This volume contains new convergence and/or complexity results for several other discretizations, including nonmatching grids [Le Tallec–Sassi–Vidrascu], nonconforming elements [Brenner, Sarkis], spectral multidomain [Azaiez–Quarteroni, Pavarino], sinc functions [Lybeck–Bowers],  $h-p$  finite elements [Oden–Patra–Feng], and the multiresolution-like “sparse grids” approach [Bungartz–Griebel–Roeschke–Zenger]. In several of these methods, the discretization and solution processes are intertwined. Related to developments in the discretizations themselves are adaptive grid refinements [Mishev, Shih–Liem–Lu–Zhou].

- *coarsened operators:*

With the non-traditional discretizations come new challenges for the derivation of appropriate operators for one or more coarsened spaces. Coarsened spaces play at least two critical roles as far as the mathematical analysis of domain decomposition methods are concerned (and perhaps others from a computer architecture

point of view). They may be used to weaken or remove the dependence of the convergence rate on the number (or size) of the subdomains, and on jumps in the coefficients of the differential equation. A coarsened unstructured grid will, in general, not be nested in the fine unstructured grid for which it is created. Therefore, the corresponding function spaces are not nested. This situation has been dealt with in multigrid theory, and is practically addressed in [Bank–Xu, Chan–Smith]. [Kornhuber–Yserentent] consider the case in which the domain itself is not resolved by the coarse grid, illustrating with a fractal-like domain. Fundamental requirements for a good coarse space and some practical examples, including for higher-order fine spaces, are given in [Widlund].

- *other preconditioner developments:*

Regarding preconditioners, as has been noted in a different context, anyone “has a perfect chance to find a better one.”<sup>1</sup> Regarding parallel preconditioners, the field is even more wide open, since serial suboptimality may be tolerated in trade-offs that favor computer architectural considerations. Preconditioner candidates appearing in these proceedings include fast summation techniques (essentially  $O(n \log n)$  dense convolutions) for the coarse grid solver required in many elliptic preconditioners [Scott] and preconditioning a Krylov method by another Krylov method, applied independently within subdomains [Pernice]. A “one shot” method combines domain decomposition on the base grid with the fictitious domain (or domain imbedding) method for potential problems with irregular geometry [Glowinski–Pan–Periaux]. The boundary element formulation of the potential problem is employed in conjunction with a two-level BPS-type preconditioner with optimal results for the model problem in [Steinbach]. The rich theory of wire-basket preconditioners continues to be undergirded, most recently by [Pavarino, Shao]. In addition, “probing” for interface preconditioner blocks has received new experimental attention in the context of large coefficient variations [Giraud–Tuminaro].

- *non-PDE problems:*

One-dimensional problems under shooting are interpreted as domain decomposition methods in [Lai]. Calling the partitioning of a search space for the roots of an algebraic problem for a system of nonlinear equations a form of domain decomposition, [Mejzlik] proposes a generalized bisection root finder.

- *implementations and architectural considerations:*

Domain decomposition leads to a truncated form of nested dissection ordering in [Lin, Mehrabi–Brown]. This purely Schur form of parallelism on a distributed memory system turns out to be competitive with conventional finite element software on vector supercomputers. Implementations of capacitance matrix and box- and strip-based domain decomposition preconditioners are compared on shared memory parallel and superscalar architectures in [Ciarlet]. Parallel implementations of Krylov-Schwarz domain decomposition algorithms on networks of high-performance workstations are introduced in [Bjørstad–Coughran–Grosse]. Their practical discussion of the capabilities and limitations of networks will help orient researchers who are contemplating this seemingly cost-effective environment. The coarse grid problem, though key to the optimal convergence rates achievable by domain decomposition methods, is a bottleneck to parallelism on most realizable architectures, as

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<sup>1</sup>G. Strang, *Introduction to Applied Mathematics*, Wellesley-Cambridge Press, 1986.

it also is in multigrid. Interesting attempts to overcome this manifestation of the “conservation of tsuris”<sup>2</sup>, are featured in [Farhat–Chen, Roux–Tromeur–Dervout], which also consider the algorithmically related problem of multiple right-hand sides in a Krylov method.

- *partitioning tools and environments:*

As solvers mature, and the dependence of their convergence rates and parallel efficiencies on the partitioning is exposed, the partitioning itself is becoming a significant research interest. A mathematically elegant solution to the partitioning problem is recursive spectral bisection [Pothen], in which a small number of eigenvectors of the Laplacian matrix of the graph of the grid are used to find the partitioning cuts. A compiler for parallel finite element methods with domain-decomposed unstructured meshes is demonstrated in [Shewchuk–Ghattas]. The paper [Mu–Rice] argues for domain decomposition as a framework for the design of object-oriented software, while [Chrisochoides–Fox–Thompson] present a problem-solving environment for mapping subdomains and generating grids in parallel.

- *applications:*

The technologically important subject of semiconductor device simulation had not taken its rightful place alongside computational fluid dynamics and computational structural mechanics in domain decomposition proceedings until the current developments in [Bjørstad–Coughran–Grosse, Coomer–Graham, Giraud–Tuminaro, Micheletti–Quarteroni–Sacco]. The stiff problems of structural mechanics, caused by aspect ratios and anisotropic material properties that are extreme even in model problems have steadily driven domain decomposition theory, with the latest installment in [Le Tallec–Mandel–Vidrascu]. The physically smooth blending between the domains of applicability of Euler and Navier-Stokes models that occurs in the physical world is mimicked in the “ $\chi$ ” approach of [Arina–Canuto]. The primarily internal flow realm of incompressible Navier-Stokes is considered in [Jacobs–Mousseau–McHugh–Knoll, Raspo–Ouazzani–Peyret, Vozovoi–Israeli–Averbuch]. Specific applications of an expanding jet [Ku–Gilreath–Raul–Sommerer], detonation combustion [Cai], external aerodynamics [Cai–Gropp–Keyes–Tidriri, Hsiao–Marcozzi–Zhang] and the shallow water equations in geophysical contexts [Cai–Navon, Chefter–Chu–Keyes] are also taken up herein. The elliptically-dominated nonlinear Poisson-Boltzmann equation is solved by a variety of multigrid and domain decomposition-based methods in [Holst–Saied], one of the few settings apart from the Poisson problem in which these two families of methods have been carefully compared. Perhaps the most novel application area for domain decomposition relative to previous proceedings is to the Bellman equations [Camilli–Falcone–Lanucara–Seghini], where the state space of the relevant PDE may have dimension much larger than three.

For the convenience of readers coming recently into the subject of domain decomposition methods, a bibliography of previous proceedings is provided below, along with some major recent review articles and related special interest volumes. This list is about twice as large as could have been offered last year, and yet, will probably be embarrassingly incomplete by the time it is published. (No attempt

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<sup>2</sup>from the Yiddish for “trouble”

has been made to supplement this list with the larger and closely related literature of multigrid and general iterative methods, except for the book by Hackbusch, which has a significant domain decomposition component.)

- (1) T. F. Chan and T. P. Mathew, *Domain Decomposition Algorithms*. Acta Numerica, 1994, pp. 61-143.
- (2) T. F. Chan, R. Glowinski, J. Périaux and O. B. Widlund, eds., *Proc. Second Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Los Angeles, 1988), SIAM, Philadelphia, 1989.
- (3) T. F. Chan, R. Glowinski, J. Périaux, O. B. Widlund, eds., *Proc. Third Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Houston, 1989), SIAM, Philadelphia, 1990.
- (4) C. Farhat and F.-X. Roux, *Implicit Parallel Processing in Structural Mechanics*, Computational Mechanics Advances **2**, 1994, pp. 1-124.
- (5) R. Glowinski, G. H. Golub, G. A. Meurant and J. Périaux, eds., *Proc. First Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Paris, 1987), SIAM, Philadelphia, 1988.
- (6) R. Glowinski, Yu. A. Kuznetsov, G. A. Meurant, J. Périaux and O. B. Widlund, eds., *Proc. Fourth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Moscow, 1990), SIAM, Philadelphia, 1991.
- (7) W. Hackbusch, *Iterative Methods for Large Sparse Linear Systems*, Springer, Heidelberg, 1993.
- (8) D. E. Keyes, T. F. Chan, G. A. Meurant, J. S. Scroggs and R. G. Voigt, *Proc. Fifth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (Norfolk, 1991), SIAM, Philadelphia, 1992.
- (9) D. E. Keyes, Y. Saad and D. G. Truhlar, eds., *Domain-based Parallelism and Problem Decomposition Methods in Science and Engineering*, SIAM, Philadelphia, 1995 (to appear).
- (10) P. Le Tallec, *Domain Decomposition Methods in Computational Mechanics*, Computational Mechanics Advances **2**, 1994, pp. 121-220.
- (11) A. Quarteroni, J. Périaux, Yu. A. Kuznetsov and O. B. Widlund, eds., *Proc. Sixth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Como, 1992), AMS, Providence, 1994.
- (12) B. F. Smith, P. E. Björstad and W. D. Gropp, *Domain Decomposition: Parallel Multilevel Algorithms for Elliptic Partial Differential Equations*, Cambridge Univ. Press, Cambridge, 1995 (to appear).
- (13) J. Xu, *Iterative Methods by Space Decomposition and Subspace Correction*, SIAM Review **34**, 1991, pp. 581-613.

The technical direction of the Seventh International Conference on Domain Decomposition Methods in Scientific and Engineering Computing was provided by a scientific committee consisting of: James H. Bramble, Tony F. Chan, Raymond C. Y. Chin, Peter J. Deuffhard, Roland Glowinski, Gene H. Golub, David E. Keyes, Yuri A. Kuznetsov, Jacques Périaux, Alfio Quarteroni, Olof B. Widlund, and Jinchao Xu. Local organization was undertaken by the following members of the faculty of The Pennsylvania State University: Douglas N. Arnold, Jerry L. Bona, Min Chen, Ali Haghihat, Jie Shen, Simon J. Tavener, and Jinchao Xu (Chair). The scientific and organizing committees, together with all attendees, are grateful

to the following agencies, organizations, corporations, and departments for their financial and logistical support of the conference: the National Science Foundation (DMS-9301980), U. S. Department of Energy (DE-FG0293ER25184), the Institute for Mathematics and its Applications at the University of Minnesota, France's GAMNI, the IBM Corporation, Cray Research Corporation, and the Applied Research Lab, the College of Engineering, the Department of Mathematics, and the Eberly College of Science at The Pennsylvania State University.

The American Mathematical Society is publishing the proceedings of the International Conference on Domain Decomposition Methods for the second consecutive time. The editors are very grateful to Donna L. Harmon of AMS for her pacing and her patience, and to Ling Shen, a doctoral candidate in the Department of Mathematics at The Pennsylvania State University for her assistance with reformatting several papers received in nonconforming styles. A single thorough round of editing, followed by the authors assuming final responsibility for the revised camera ready copy, seems to permit a good balance between the quality of the proceedings and the promptness of its publication.

Our families graciously forsook many weekends together for this collection and are trusting, as are we, in a useful shelf life.

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September 1994