Thirteenth International Conference on Domain Decomposition Methods
Editors: N. Debit, M.Garbey, R. Hoppe, J. Périaux, D. Keyes, Y. Kuznetsov

48 Optimized Interface Conditions for Sedimentary Basin Modeling

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Why DDM for basin modeling ?

Basin modeling aims at reconstructing the time evolution of a sedimentary basin in order to make quantitative predictions of geological phenomena leading to oil accumulations. It accounts for porous medium compaction, heat transfer, hydrocarbon formation and migration. Recent evolutions of basin simulators have contributed to improve the treatment of geological discontinuities such as faults and salt domes. Faults divide the basin into blocks which slide between themselves. They may be a preferential path or in opposite a barrier for hydrocarbons migration. A salt is an impervious medium and becomes a trap for hydrocarbon. CERES is an advanced prototype of $2D$ sedimentary basin tool that can handle non-vertical faults and salt or mud tectonics (figure 1). Domain decomposition methods provide a way to solve the

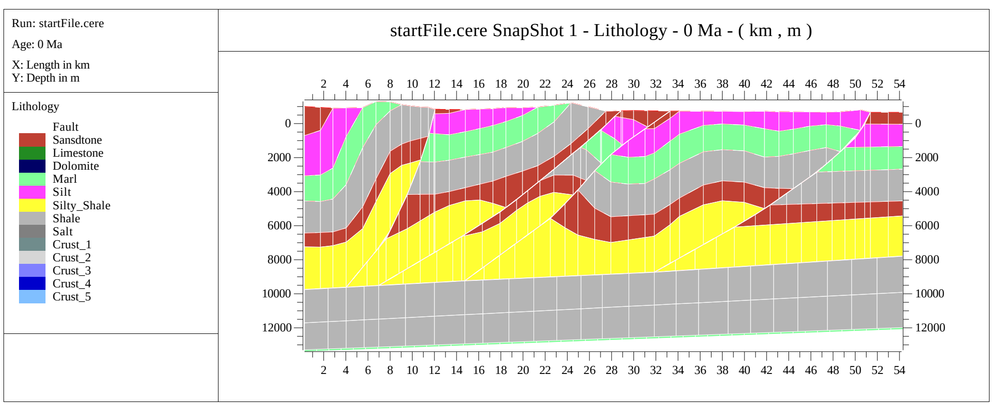

Figure 1: CERES 2D real basin

equations on the complex geometries considered, naturally defined as a set of adjacent sliding blocks and faults.

Following the work of [NR95], [NRdS94], [JNR01], we use nonoverlapping techniques and study several interface conditions, namely Robin type conditions.

The paper is organized as follows. First, the physical models and the governing equations taken into account are reviewed. Then the DDM are presented on a simpler equation in which we find the main characteristics of the problem. The optimized interface conditions are detailed for this equation. Finally, numerical results are shown.

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Models and governing equations

In the blocks, the model accounts for the porous medium compaction, erosion, heat transfer, hydrocarbon formation and migration. The equations are mass conservation of solid and fluids (water,oil,gas) coupled with the Darcy's law and a compaction law. The faults have a constant porosity, but the permeability of the faults evolves in time. We consider only incompressible multiphase flows.
To present the DD method we consider a simplified basin model where geometry does not evolve in time. Using an IMPES (IMplicit Pressure, Explicit Saturation) scheme, we first solve a parabolic pressure equation and then update explicitly the phase saturations.
After time discretization, the pressure equation is then roughly written as follows :

$$\mathcal{L}(P) = \frac{\alpha}{\Delta t} P + div(-K\overrightarrow{grad}P) = f \tag{1}$$

where P is the pressure, α the compressibility of the porous medium and K the intrinsic permeability tensor of the porous medium divided by the fluid viscosity. The permeability depends heavily on the lithology under consideration. The contrast in the lithologies can induce a discontinuity of the permeability tensor of several orders of magnitude (up to six orders).
Moreover we have to deal with subdomains of various size, block width is about 10 km while fault width is about 10 m.

The DDM for the pressure equation

Our goal is to find a domain decomposition method robust enough to deal with the strong discontinuities that can arise along and across an interface between two subdomains and whose behavior is not ruined by small subdomains. To cope with these difficulties, we introduce a Robin type interface condition whose coefficients are computed in order to optimize the convergence rate of the Additive Schwarz method (ASM for short).
We consider the parabolic linear equation (1) with strongly discontinuous coefficients K. We cut the domain into nonoverlapping subdomains Ω_i and solve the equation in each subdomain. In the framework of this paper, we weigh up only matching grid but the approach is extended to non-matching grid as see on figure 1.
Pressure and flux continuity between two subdomains Ω_1 and Ω_2 are expressed as Robin conditions on the interface Γ:

$$\alpha_2 P_1 + \beta_2 (K\overrightarrow{grad}P)_1.\vec{n_1} = \alpha_2 P_2 - \beta_2 (K\overrightarrow{grad}P)_2.\vec{n_2} \quad \text{on } \Gamma \tag{2}$$

$$\alpha_1 P_2 + \beta_1 (K\overrightarrow{grad}P)_2.\vec{n_2} = \alpha_1 P_1 - \beta_1 (K\overrightarrow{grad}P)_1.\vec{n_1} \quad \text{on } \Gamma \tag{3}$$

where α_i, β_i are real such that $\alpha_1\beta_2 + \alpha_2\beta_1 \neq 0$ and $\alpha_i\beta_i > 0$.
The idea is to find the coefficients (α_i, β_i) which allow a fast convergence of DD algorithm, namely ASM with the boundary condition (2) in Ω_1 and boundary condition (3) in Ω_2. This has been introduced by Nataf and co-author for convection-diffusion equation [NRdS94] [NR95].

To compute the Robin coefficients, we successively address two main difficulties. First, we consider the case of two subdomains with a jump of permeability across the interface. Secondly, we deal with two subdomains separated by a small fault.

Robin conditions for two unbounded subdomains

We consider two unbounded subdomains Ω_1, Ω_2 with an interface Γ. The subdomains have homogeneous permeability K_1 in subdomain Ω_1 and K_2 in Ω_2. The computation of Robin co-

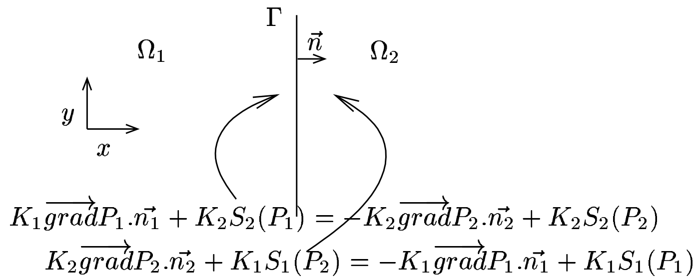


Figure 2: Optimal interface condition for 2 domains

efficients is based on the approximation of the Optimal Interface Condition. Optimal Interface Conditions are conditions which ensure convergence of ASM in 2 iterations for a decomposition into 2 subdomains. They extend the Artificial Boundary Condition and keep the idea of “packing the neighboring domain problem on the interface Γ”. To do so, let us define the classical Steklov-Poincaré operator S_i associated to Ω_i :

$$S_i : P_{|\Gamma} \longrightarrow \overrightarrow{grad} P_i . \vec{n_i}_{|\Gamma} \qquad \text{where } P_i \qquad \begin{cases} \mathcal{L}_i(P_i) & = 0 \text{ in } \Omega_i \\ P_i & = P \text{ on } \Gamma \end{cases}$$

We can write the Optimal Interface Condition as follows, for Ω_1 :

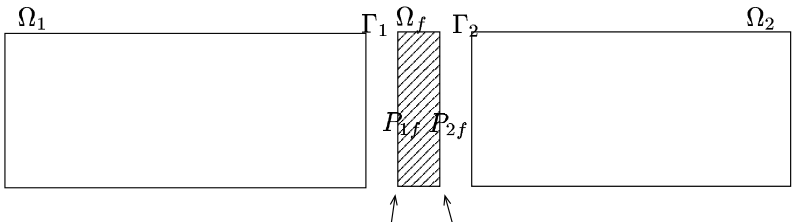
$$K_1 \overrightarrow{grad} P_1 . \vec{n_1} + K_2 S_2(P_1) = -K_2 \overrightarrow{grad} P_2 . \vec{n_2} + K_2 S_2(P_2)$$

For Ω_2, we have :

$$K_2 \overrightarrow{grad} P_2 . \vec{n_2} + K_1 S_1(P_2) = -K_1 \overrightarrow{grad} P_1 . \vec{n_1} + K_1 S_1(P_1)$$

The contribution of subdomain Ω_2 appears through S_2. The jump of permeability K is found in the two terms K_1, K_2.
To find the OIC, we need to make explicit the Steklov-Poincaré operator . Therefore we perform a Fourier transform with respect to y of equation (1) and we solve exactly the obtained equation which only depends on the x variable. The expression of Λ_i, symbol of S_i, are not polynomial in k, dual variable with respect to y. The Steklov-Poincaré operator and so the OIC are not local in space and must be approximated.
In order to obtain the Robin boundary condition, two constant approximations have been considered, giving the following coefficients:

- taking a Taylor expansion of Λ_i, we obtain: $(\alpha_i, \beta_i) = (K_i \omega_i, 1)$,
- we perform the minimization of the convergence rate on a frequency slot (k_{min}, k_{max}). This can be done only in the homogeneous case. We have the following expression:



$$
\begin{aligned}
K_f \overrightarrow{grad} P_{1f} . \vec{n_{1f}} + K_1 S_1(P_{1f}) &= -K_1 \overrightarrow{grad} P_1 . \vec{n_1} + K_1 S_1(P_1) \\
K_f \overrightarrow{grad} P_{2f} . \vec{n_{2f}} + K_2 S_2(P_{2f}) &= -K_2 \overrightarrow{grad} P_2 . \vec{n_2} + K_2 S_2(P_2)
\end{aligned}
$$

Figure 3: OIC for the fault

$(\alpha_i \ , \beta_i) = (K_i \sqrt{\sqrt{\omega_i^2 + k_{min}^2} \sqrt{\omega_i^2 + k_{max}^2}} \ , 1)$. This leads to the geometric average of the operator at the two extreme frequencies (k_{min}, k_{max}).

Although optimized in the homogeneous case, these conditions are efficient in the heterogeneous case. Indeed the coefficient α_i is a good approximation of S_i.
These last conditions have been implemented and give good results for subdomains with more or less similar size [WFS96]. However in the case of two blocks separated by a fault, the results need to be improved.

Robin conditions for two unbounded domains and one fault

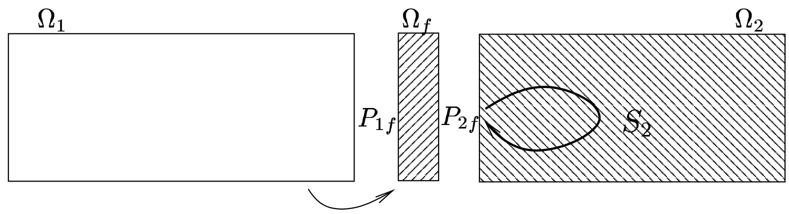
Now, we examine the case of three subdomains : we have two unbounded subdomains Ω_1, Ω_2 with a small fault Ω_f. The permeability of each subdomain is uniform : K_1 in Ω_1, K_2 in Ω_2 and K_f in Ω_f. We first set up the interface condition on the fault boundaries (see figure 3). As for a given boundary, the fault has only one neighbor, so we can define the Optimal Interface Condition as previously, where S_i is the Steklov-Poincaré operator associated to domain Ω_i.
Next we set up the OIC on the blocks boundaries. Assume, we are on the boundary of Ω_1 to find the OIC (see figure 4). The OIC uses S_{f+2} the Steklov-Poincaré operator associated to $\Omega - \Omega_1$:

$S_{f+2} : P_{1f} \longrightarrow \overrightarrow{grad} P . \vec{n_f}(P_{1f})$, where P is solution to

$$
\begin{cases}
\mathcal{L}_f(P) & = 0 \ \text{in} \ \Omega_f \quad ; \quad \mathcal{L}_2(P) \quad = 0 \ \text{in} \ \Omega_2 \\
P & = P_{1f} \ \text{on} \ \Gamma_1 \quad ; \quad P \ \text{and} \ K \overrightarrow{grad} P . \vec{n} \ \text{continuous on} \ \Gamma_2
\end{cases}
$$

Although more complex, the Steklov-Poincaré operator is determined as previously performing a Fourier transform with respect to y. This operator S_{f+2} depends on the permeabilities K_2 in domain Ω_2, K_f in the fault and the fault width. As before the operators S_i, S_{i+f} are non-local in space, so we compute a polynomial approximation:

- we take the OIC for a frequency k_0,
- it is difficult to optimize the convergence rate, so we keep the idea to approximate the operator by the geometric average of two extreme values:



$$K_1\overrightarrow{grad}P_1.\vec{n_1} + K_f S_{f+2}(P_1) = \quad -K_f\overrightarrow{grad}P_f.\vec{n_f} + K_f S_{f+2}(P_f)$$

Figure 4: OIC for the subdomain Ω_1

Fault boundary condition:$(\alpha_{f\,i}\ ,\beta_{f\,i}) = (K_i\sqrt{\Lambda_i(k_{min})\Lambda_i(k_{max})}\ ,1)$;
Block boundary condition: $(\alpha_i\ ,\beta_i) = (K_f\sqrt{\Lambda_{f+i}(k_{min})\Lambda_{f+i}(k_{max})}\ ,1)$.

In the following, we denote by OIC, Robin conditions with these last (α_i, β_i) coefficients.

Numerical results

The approach is then extended to a real basin model (CERES 2D) which accounts for porous medium sedimentation, compaction, erosion and blocks displacements along faults. In CERES 2D, we have a discontinuity jump of the permeability K along the interface, so the Robin coefficient α_i is computed locally on each edge. The interface problem is solved with the GMRES algorithm. The unknowns are $H_1 = \alpha_2 P_1 + \beta_2 F_1$, $H_2 = \alpha_1 P_1 + \beta_1 F_2$, .

$$\begin{array}{l}\mathcal{H}_{12} \quad : \quad H_1 \to \alpha_1 P_1 + \beta_1 F_1 \\ \quad \text{with } F_1 = (-K_1\overrightarrow{grad}P_1.\vec{n_1})\end{array} \quad \text{and} \begin{cases}\mathcal{L}_1(P_1) & = f \ \text{ in } \Omega_1 \\ BC & \text{ on } \partial\Omega_1/\Gamma \\ \alpha_2 P_1 + \beta_2 & (K_1\overrightarrow{grad}P_1.\vec{n_1}) = H_1 \ \text{ on } \Gamma\end{cases}$$

The equations are
Pressure and Flux continuity

$$\begin{cases}\mathcal{H}_{12}(H_1) - H_2 & = 0 \\ H_1 - \mathcal{H}_{21}(H_2) & = 0\end{cases}$$

Numerical results show the good behavior of the Robin interface conditions. Comparisons with the Dirichlet-Neumann conditions illustrate the robustness and the good convergence rate of DD algorithms such as additive Schwarz method, possibly accelerated by GMRES.

Mesh refinement

We consider a synthetic basin (figure 5) composed of two heterogeneous blocks with K_1, K_2 permeability and a fault with K_f permeability. We want to study the influence of vertical mesh subdivision: each row is subdivided in 2 then 4, so the number of interface unknowns is growing. Moreover, the permeability of the fault is either: pervious, impervious or an average of the two neighbouring block cells (variable for short).
In Figure 6, we report the number of DDM iterations as a function of the number of interface unknowns for different fault permeability. The dotted lines show the Dirichlet-Neumann

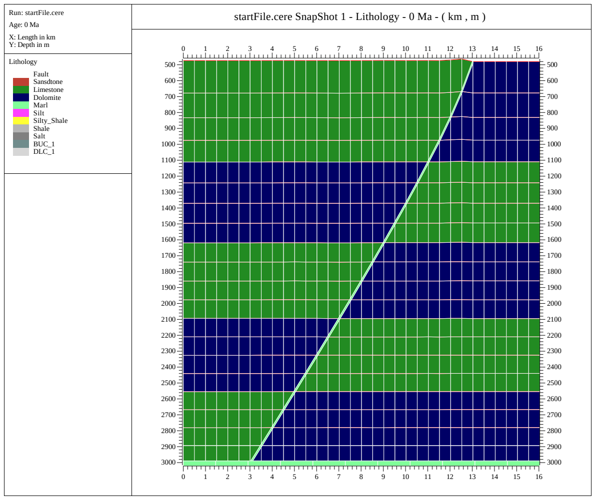


Figure 5: Synthetic basin

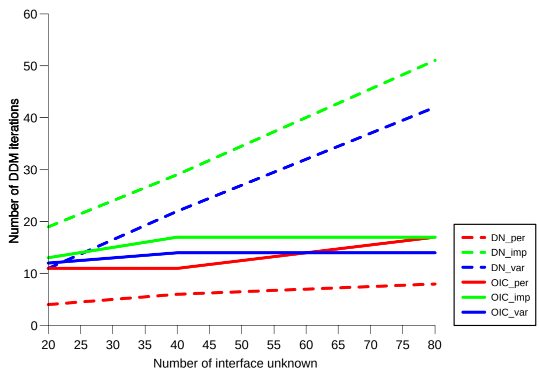


Figure 6: Number of DDM iterations

condition, Dirichlet on blocks, Neumann on fault,(DN for short) and the solid lines show the Robin condition. The Dirichlet-Neumann interface condition is very competitive for pervious fault but not for the other cases. The Robin conditions converge well; all curves correspond to 10 iterations of DDM. The number of DDM iterations does not increase too much with the number of interface unknowns for OIC. The behavior of OIC is regular for all spreading of fault permeability.

Time evolution

We can study the time evolution of DDM since the number of unknowns increases through time as new layers of sediments are deposited. Each layer corresponds to a row of homogeneous cells. On the following figure 7, a synthetic basin is composed of two heterogeneous

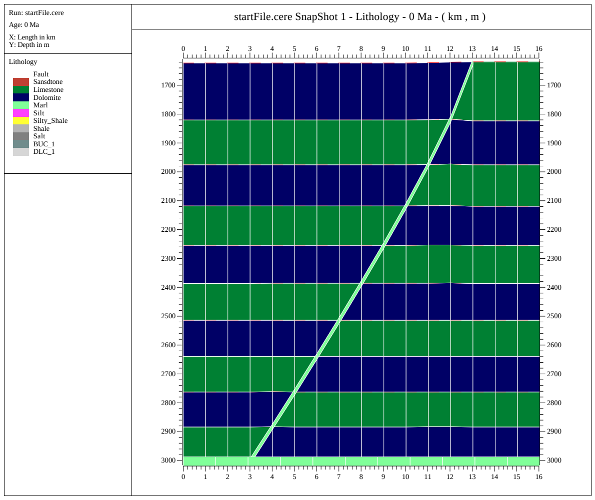


Figure 7: Synthetic basin

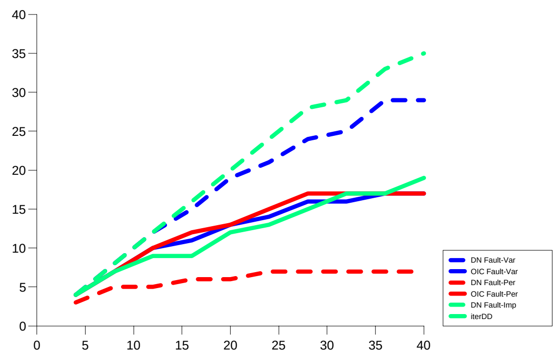


Figure 8: Number of DDM iterations

blocks and one fault. Each block has alternated and shift row of pervious-impervious medium. The fault permeability is successively: pervious, impervious or variable.

In Figure 8, we report the number of DDM iterations as a function of the number unknowns. The dotted lines show DN condition and the solid lines show OIC. The number of DDM iterations grows slowly with the number of unknowns for the OIC. Here again, OIC show robustness regarding subdomain heterogeneities.

Conclusion

We introduced a domain decomposition method applied to sedimentary basin modeling. The DDM is robust enough to overcome high jump of heterogeneity, up to 6 orders and various sizes of subdomains. To do this, we have chosen a nonoverlapping ASM with Optimal Interface condition, Robin type. The interface problem is solved with a GMRES algorithm. Despite the good results obtained, the fault is still a very small subdomain compared to the blocks. Therefore it seems promising to consider one dimensional fault [Fla01]. The fault model is then included in the interface condition between two blocks. We wish to improve the non-matching approach so as to win in flexibility and to have less interface unknowns. Another improvement is to extend the DDM to a "fully implicit" discretization scheme for multiphase flow. DDM will be applied to a system of pressure and saturation variables.

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